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THE CLASSICAL ANALOG OF THE AHARONOV-BOHM EFFECT

Abstract. *This article is dedicated to the classical analogue of the Aharonov-Bohm effect, which consists in the fact that a charged particle moving near a current-carrying solenoid experiences a force dependent on the vector potential of the magnetic field. The authors demonstrate that this effect can be explained through classical electrodynamics, taking into account that the vector potential induces an additional force.*

Анотація. *Ця стаття присвячена класичному аналогу ефекту Ааронова-Бома, який полягає в тому, що на заряджену частинку, що рухається поблизу соленоїда зі струмом, діє сила, яка залежить від векторного потенціалу магнітного поля. Автори демонструють, що цей ефект можна пояснити за допомогою класичної електродинаміки, враховуючи, що векторний потенціал індукує додаткову силу.*

Key words: *Aharonov-Bohm effect, vector potential, classical electrodynamics, solenoid, charged particle.*

Ключові слова: *ефект Ааронова-Бома, векторний потенціал, класична електродинаміка, соленоїд, заряджена частинка.*

According to classical electrodynamics, on an electric charged q particle exert force that is determined by the electric $\vec{E}$ and magnetic $\vec{B}$ fields at the point where the charge is located $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$. Note that since the text contains expressions related to quantum mechanics, we here and further use the Gaussian system of units. Consequently, these fields are included in Newton's equation and determine the movement of the charge.

On the other hand, in quantum mechanics, the Schrödinger equation, which describes the behavior of a charge in electromagnetic fields, includes not the electric and magnetic fields themselves, but their potentials - the scalar potential - φ and the vector potential - $\mathbf{A}$

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\left(\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A}\right)^2 + e\varphi .$$

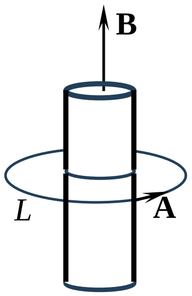

Fig.1

Let's now consider the behavior of an electron when passing near an infinitely long solenoid with a current (see Fig. 1). The magnetic $\mathbf{B}$ field exists only inside the solenoid, outside the solenoid there is no magnetic field. In the case of a direct current flowing in a solenoid, the electric field is absent too.

In quantum mechanics, it is necessary to consider that both inside and outside (beyond the solenoid), there exists a vector potential $\mathbf{A}$ created by the current-carrying turns. In an interference experiment, when an electron passes near a coil, there is a phase accumulation in the electron's wave, and the interference pattern changes compared to the case when there is no current in the solenoid consequently there is no magnetic field (see for example [1]). This is the Aharonov-Bohm effect - the influence of an electromagnetic field on a charged particle even in those regions where the electric and magnetic fields are zero, but the vector potentials are not zero [2, 3].

In the classical case, we consider a solenoid with a current increasing linearly in time and a correspondingly increasing magnetic field inside the solenoid. Outside the solenoid, there is still no magnetic field. In this case, a time-dependent magnetic flux $\Phi(t) = \int_S \mathbf{B} d\mathbf{s}$ flows through the surface S stretched on the contour L (see Fig. 1), and consequently an electric field exists along the contour.

Let the magnetic field of the solenoid depend on time as $B(t) = B_0\, t/\tau,$ (where τ is a certain time constant). Then according to Maxwell's equation written in integral form $\oint \mathbf{E} d\mathbf{r} = -4\pi / c d\Phi(t) / dt$ the electric field outside the solenoid will be equal to $E(r > R) = -\frac{1}{2c}\frac{B_0}{\tau}\frac{R^2}{r}$, where R is the radius of the solenoid.

Recall that the electromagnetic theory is local, i.e., there is no magnetic field outside the solenoid and the coil is not charged. The question arises, what is the cause of the electric field, and thus the force acting on the charge, at any point outside the solenoid? Just like in the Aharonov-Bohm effect, the answer lies in the fact that outside the solenoid, there exists a vector potential, more precisely, its component tangential to the contour $A = BR^2/2r$. At the same time, there is no magnetic field outside the solenoid – $\boldsymbol{B} = rot\boldsymbol{A} = \mathbf{0}$. However, in the case of time-dependent magnetic fields, there is an electric field, and according to $\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}$, it precisely corresponds to what is derived from the integral form of Maxwell's equation $E\left(r > R\right) = -\frac{1}{2c}\frac{B_0}{\tau}\frac{R^2}{r}$.

Thus, within a local approach (Maxwell's equations in the form of partial differential equations), it is possible to find the force acting on a charge by using the vector potential.

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